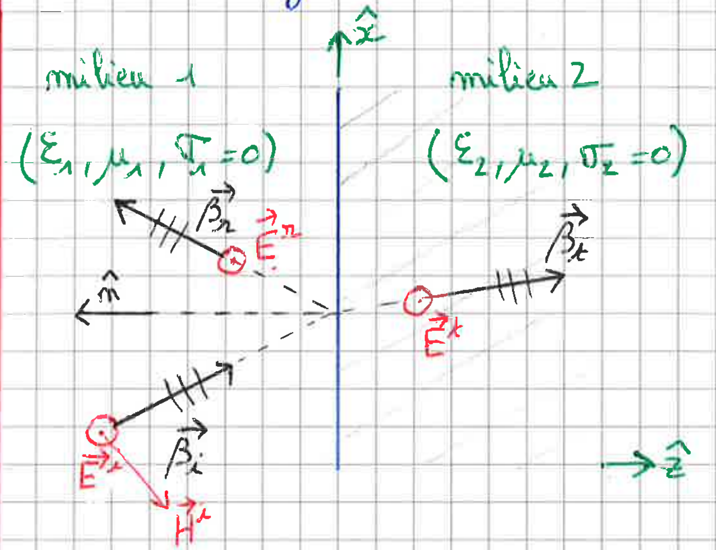
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

d _ Coefficients de Fresnel

C40 _ Interface TE

plan d'incidence = plan contenant $\vec{\beta}_i$ et $\hat{n}$



= polarisation perpendiculaire
- TE_z (car $E_z = 0$)
- S
- horizontale

$\vec{\beta}_i = \beta_{x_i}\hat{x} + \beta_{z_i}\hat{z}$ avec $\boxed{\beta_{x_i}^2 + \beta_{z_i}^2 = \omega^2\mu_1\varepsilon_1}$ RD milieu ①

Champ incident : $\vec{E}^i = \hat{y}\, E_0\, e^{-j(\beta_{x_i}x + \beta_{z_i}z)}$

$$\vec{H}^i = \frac{1}{\omega\mu_1}(\vec{\beta}_i \times \vec{E}^i) = \frac{E_0}{\omega\mu_1}(-\hat{x}\beta_{z_i} + \hat{z}\beta_{x_i})\, e^{-j(\beta_{x_i}x + \beta_{z_i}z)}$$

$\vec{E}^t$, $\vec{E}^r$ aussi suivant $\hat{y}$ car les charges liées dans ② ne bougent que en $\hat{y}$.

C41 _ Coefficients

Champ réfléchi : $\vec{E}^r = \hat{y}\, E_0\, \Gamma_\perp\, e^{-j(\beta_{x_r}x - \beta_{z_r}z)}$ $\Gamma_\perp$ = coeff de réflexion $\in \mathbb{C}$

$\vec{\beta}_r = \beta_{x_r}\hat{x} - \beta_{z_r}\hat{z}$, $\boxed{\beta_{x_r}^2 + \beta_{z_r}^2 = \omega^2\mu_1\varepsilon_1}$ RD milieu ①

$$\Rightarrow \vec{H}^r = \frac{1}{\omega\mu_1}(\vec{\beta}_r \times \vec{E}_r) = \frac{\Gamma_\perp E_0}{\omega\mu_1}(\beta_{z_r}\hat{x} + \beta_{x_r}\hat{z})\, e^{-j(\beta_{x_r}x - \beta_{z_r}z)}$$

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Champ transmis $\vec{E}^t = \hat{y}\, T_\perp E_0\, e^{-j(\beta_{x_t} x + \beta_{z_t} z)}$ $T_\perp$ coef de transmission

$\vec{\beta}_t = \beta_{x_t}\hat{x} + \beta_{z_t}\hat{z}$, $\boxed{\beta_{x_t}^2 + \beta_{z_t}^2 = \omega^2 \mu_2 \varepsilon_2}$ RD milieu ②

$$\Rightarrow \vec{H}^t = \frac{E_0 T_\perp}{\omega\mu_2}\left(-\hat{x}\beta_{z_t} + \cancel{\hat{y}}\,\hat{z}\,\beta_{x_t}\right) e^{-j(\beta_{x_t} x + \beta_{z_t} z)}$$

CL à $z=0$: $\vec{E}_{tan}$ et $\vec{H}_{tan}$ sont continus ($\vec{J}_s = 0$)

$$\Rightarrow \begin{cases} E_y^i + E_y^r = E_y^t \\ H_x^i + H_x^r = H_x^t \end{cases} \Rightarrow \cancel{E_0} e^{-j\beta_{x_i} x} + \cancel{E_0}\, r_\perp e^{-j\beta_{x_r} x} = \cancel{E_0} T_\perp e^{-j\beta_{x_t} x} \quad \forall x !$$

$$\Rightarrow \boxed{\beta_{x_i} = \beta_{x_r} = \beta_{x_t}}$$ continuité de phase

$$\Rightarrow \boxed{1 + r_\perp = T_\perp} \quad (1)$$

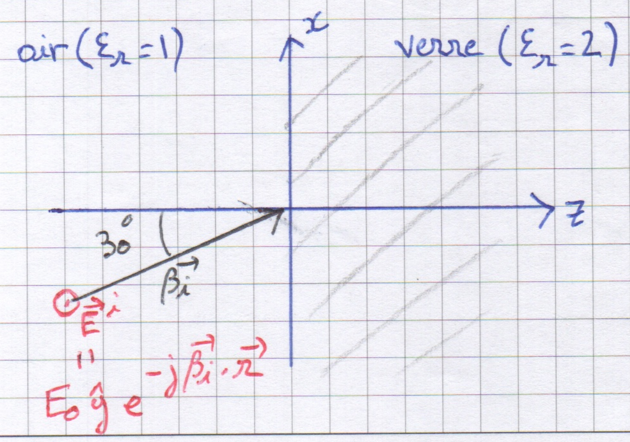
$$H_x^i + H_x^r = H_x^t \Rightarrow -\frac{E_0\beta_{z_i}}{\omega\mu_1} e^{-j\beta_{x_i} x} + \frac{E_0\beta_{z_r}}{\omega\mu_1} r_\perp e^{-j\beta_{x_r} x} = -\frac{E_0\beta_{z_t}}{\omega\mu_2} T_\perp e^{-j\beta_{x_t} z}$$

$$\Rightarrow \boxed{1 - r_\perp = \underbrace{\frac{\mu_1}{\mu_2}\frac{\beta_{z_t}}{\beta_{z_i}}}_{y_\perp} T_\perp} \quad (2)$$

(1) et (2) $\Rightarrow$ $$\boxed{T_\perp = \frac{2}{1+y_\perp} \qquad r_\perp = \frac{1-y_\perp}{1+y_\perp} \qquad y_\perp = \frac{\mu_1}{\mu_2}\frac{\beta_{z_t}}{\beta_{z_i}}}$$

coefficients de Fresnel de reflection et transmission

C42 - Exemple



$\rightarrow$ Trouver $\vec{E}^r$, $\vec{E}^t$

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(i) $\vec{\beta_i} = \beta_0 \hat{\beta_i}$ avec $\beta_0 = \omega\sqrt{\mu_0\varepsilon_0} = \omega/c_0$

$= \beta_0(\sin 30° \hat{x} + \cos 30° \hat{z})$

$\Rightarrow \vec{\beta_i} = \frac{\beta_0}{2}\hat{x} + \beta_0\frac{\sqrt{3}}{2}\hat{z}$

$\vec{\beta_r} = \frac{\beta_0}{2}\hat{x} - \beta_0\frac{\sqrt{3}}{2}\hat{z}$

$\vec{\beta_t} = \frac{\beta_0}{2}\hat{x} + \beta_{z_t}\hat{z}$ $\quad \beta_{z_t}$? $\Rightarrow$ on utilise la RD dans 2

$\Rightarrow \beta_{z_t} = \sqrt{\omega^2\mu_0(2\varepsilon_0) - \beta_0^2\sin^2 30°} = \sqrt{2\beta_0^2 - \frac{1}{4}\beta_0^2} = \sqrt{\frac{7}{4}}\beta_0$

(ii) $y_\perp = \frac{\mu_0}{\mu_0}\frac{\beta_0\sqrt{7}/2}{\beta_0\sqrt{3}/2} = \sqrt{\frac{7}{3}} = 1.53 \Rightarrow \Gamma_\perp = \frac{1-y_\perp}{1+y_\perp} = -0.209$

$T_\perp = 1 + \Gamma_\perp = 0.791$

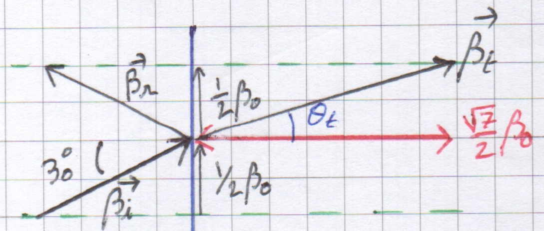
(iii) $\begin{cases} \vec{E}^i = E_0\hat{y}\, e^{-j\beta_0(\frac{1}{2}x + \frac{\sqrt{3}}{2}z)} \\ \vec{E}^r = (-0.209)E_0\hat{y}\, e^{-j\beta_0(\frac{1}{2}x - \frac{\sqrt{3}}{2}z)} \\ \vec{E}^t = \hat{y}\,(0.791)E_0 e^{-j\beta_0(\frac{1}{2}x + \frac{\sqrt{7}}{2}z)} \end{cases}$

C43_ Lois de Snell-Descartes

$* \; \beta x_i = \beta x_r \Rightarrow \cancel{\omega}\cancel{\sqrt{\mu_1\varepsilon_1}}\sin\theta_i = \cancel{\omega}\cancel{\sqrt{\mu_1\varepsilon_1}}\sin\theta_r \Rightarrow \boxed{\theta_i = \theta_r}$

$* \; \beta x_i = \beta x_t \Rightarrow \cancel{\omega}\underbrace{\sqrt{\mu_1\varepsilon_1}}_{\cancel{\sqrt{\mu_0\varepsilon_0}}\, n_1}\sin\theta_i = \cancel{\omega}\underbrace{\sqrt{\mu_2\varepsilon_2}}_{\cancel{\sqrt{\mu_0\varepsilon_0}}\, n_2}\sin\theta_t \Rightarrow \boxed{n_1\sin\theta_1 = n_2\sin\theta_t}$

Indice de refraction : $n \stackrel{def}{=} \sqrt{\mu_r\varepsilon_r}$



$\theta_t = \sin^{-1}\left(\frac{\frac{1}{2}\beta_0}{\sqrt{\frac{7}{4}+\frac{1}{4}}}\right) = 21°$

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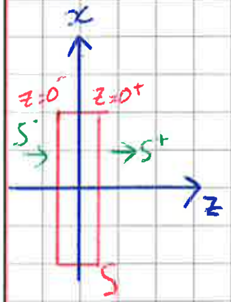
C44 - Bilan d'énergie



puissance entrante en $z=0^-$ = puissance sortante en $z=0^+$ (pas de pertes $\sigma_1=\sigma_2=0$)

$$\Rightarrow \int \frac{1}{2}\mathrm{Re}\left(\vec{E}_1\times\vec{H}_1^*\right)\cdot\hat{z}\,dS = \int\frac{1}{2}\mathrm{Re}\left\{\vec{E}_2\times\vec{H}_2^*\right\}\cdot\hat{z}\,dS$$

$$\Rightarrow \frac{1}{2}\mathrm{Re}\left[(1+\Gamma_\perp)\hat{y}\times\left(\frac{-\beta_{z_i}^*}{\omega\mu_1}\right)(1-\Gamma_\perp^*)\hat{x}\right]\cdot\hat{z}$$

$$=\frac{1}{2}\mathrm{Re}\left[T_\perp\hat{y}\times\left(\frac{-\beta_{z_t}^*}{\omega\mu_2}\right)T_\perp^*\hat{x}\right]\cdot\hat{z}$$

Pas de pertes $\Rightarrow \beta_{z_i},\beta_{z_t}\in\mathbb{R} \Rightarrow \mathrm{Re}\left[\frac{\beta_{z_i}}{\omega\mu_1}\left(1\underbrace{-\Gamma_\perp^*+\Gamma_\perp}_{\in j\mathbb{R}}+|\Gamma_\perp|^2\right)\right]=\mathrm{Re}\left[\frac{\beta_{z_t}}{\omega\mu_2}|T_\perp|^2\right]$

$$\Rightarrow \boxed{1-|\Gamma_\perp|^2=\frac{\mu_1}{\mu_2}\frac{\beta_{z_t}}{\beta_{z_i}}|T_\perp|^2=\underbrace{y_\perp|T_\perp|^2}}$$

puissance incidente

puissance réfléchie $|\Gamma_\perp|^2$

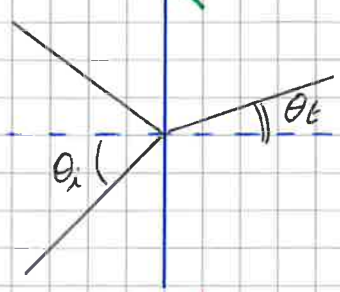
puissance transmise $y_\perp|T_\perp|^2$

ex du C42 : $y_\perp=1.53$, $\Gamma_\perp=-0.209$, $T_\perp=0.791$

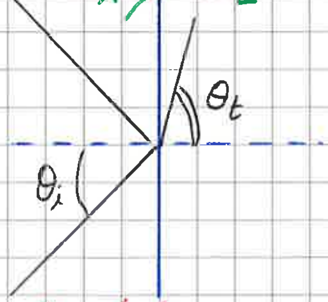
$$\Rightarrow 1-\underbrace{0.094}_{\text{9.4\% est réfléchi}}=\underbrace{0.956}_{\text{95.6\% de } P_i \text{ est transmis}}$$

C45 - Angle critique

$n_1 < n_2$ $\qquad\qquad$ $n_1 > n_2$

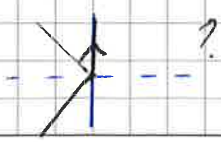


$\theta_i > \theta_t$



$\theta_i < \theta_t$

que se passe-t-il quand $\theta_t = 90°$



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On a $n_1 \sin\theta_c = n_2 \cancel{\sin 90^\circ}_{1} \Rightarrow \boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$ angle critique

Quand $\theta = \theta_c$, $\vec{\beta}_t = \beta x_t \hat{x} + 0\hat{z} = \omega_1\sqrt{\mu_2\varepsilon_2}\sin 90^\circ \hat{x} + 0\hat{z}$

$\theta < \theta_c$, $\vec{\beta}_t = \beta x_t \hat{x} + \beta_{z_t}\hat{z}$ et $\beta x_t = \omega\sqrt{\mu_2\varepsilon_2}\sin\theta_t < \omega\sqrt{\mu_2\varepsilon_2}$

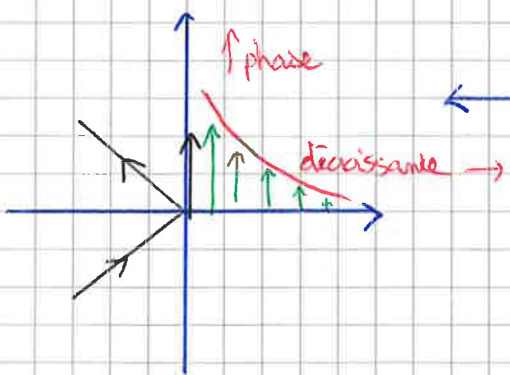
Plus intéressant, $\theta_i > \theta_c$: $\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i > 1$ → sens physique ?

$\theta_i > \theta_c \Rightarrow \beta_{x_t} > \omega\sqrt{\mu_2\varepsilon_2}$

$\Rightarrow \beta_{z_t} = \sqrt{\underbrace{\omega^2\mu_2\varepsilon_2 - \beta_{x_i}^2}_{<0}} = \sqrt{(-1)(\underbrace{\beta_{x_i}^2 - \omega^2\mu_2\varepsilon_2}_{>0})} = \pm j\alpha$

signe + → $\vec{E}_t \sim e^{-j\beta_{x_i}x}\underbrace{e^{-j(+j\alpha)z}}_{e^{\alpha z}}$ → non physique

signe − → $\vec{E}_t \sim e^{-j\beta_{x_i}x}\underbrace{e^{-j(-j\alpha)z}}_{e^{-\alpha z}}$ → OK



← onde évanescente
onde plane non uniforme
onde de surface (à droite)

Rq: $y_\perp = \frac{\mu_1}{\mu_2}\frac{\beta_{z_t}}{\beta_{z_i}} = -j(\,) \Rightarrow \Gamma_\perp = \frac{1-y_\perp}{1+y_\perp} = \frac{1+j(\,)}{1-j(\,)} \Rightarrow |\Gamma_\perp|^2 = 100\%$

réflexion interne totale